

# THE SIMPLEST DETERMINATION OF THE THERMODYNAMICAL CHARACTERISTICS OF KERR-NEWMAN BLACK HOLE

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## Abstract

In this work, generalizing our previous results, we determine in an original and the simplest way three most important thermodynamical characteristics (Bekenstein-Hawking entropy, Bekenstein quantization of the entropy or (outer) horizon surface area and Hawking temperature) of Kerr-Newman black hole. We start physically by assumption that circumference of Kerr-Newman black hole (outer) horizon holds the natural (integer) number of corresponding reduced Compton's wave length and use mathematically, practically, only simple algebraic equations. (It is conceptually similar to Bohr's quantization postulate in Bohr's atomic model interpreted by de Broglie relation.)

"Here have been deceased clouds  
dead time with history of days,  
here have been fallen rays;  
nirvana oppressed whole universe."

Vladislav Petković Dis , "Nirvana"

In this work, generalizing our previous results [1], [2], we shall reproduce and determine in the simplest way three well-known [3]-[9], most important thermodynamical characteristics (Bekenstein-Hawking entropy, Bekenstein quantization of the entropy or (outer) horizon surface area and Hawking temperature) of Kerr-Newman black hole. We shall start physically by assumption that circumference of black hole (outer) horizon holds the natural (integer) number of corresponding reduced Compton's wave length and use mathematically, practically, only simple algebraic equations. (It is conceptually similar to Bohr's quantization postulate in Bohr's atomic model interpreted by de Broglie relation.) In this work we shall use natural system of the units where speed of light, Planck constant, Newtonian gravitational constant and Boltzmann constant are equivalent to unit.

Consider a Kerr-Newman black hole with mass  $M$ , angular momentum  $J$ , electrical charge  $Q$ , outer horizon radius

$$R_+ = M + (M^2 - a^2 - Q^2)^{\frac{1}{2}} \quad (1)$$

and surface area of the outer horizon

$$A_+ = 4\pi(R_+^2 + a^2) \quad (2)$$

for

$$a = \frac{J}{M} \quad (3)$$

Suppose the following expression

$$m_{+n}R_+ = \frac{n}{2\pi}, \quad \text{for } n = 1, 2, \dots \quad (4)$$

that implies

$$2\pi R_+ = n \frac{1}{m_{+n}} = n \cdot \lambda_{r+n} \quad \text{for } n = 1, 2, \dots \quad (5)$$

Here  $2\pi R_+$  represents the circumference of the outer horizon while

$$\lambda_{r+n} = \frac{1}{m_{+n}} \quad (6)$$

represents the  $n$ -th reduced Compton wavelength of a quantum system with mass  $m_{+n}$  captured at black hole outer horizon for  $n = 1, 2, \dots$ .

Expression (5) simply means that *circumference of the black hole outer horizon holds exactly  $n$ -th reduced Compton wave lengths of quantum systems for  $n = 1, 2, \dots$* . Obviously, it is conceptually similar to well-known Bohr's quantization postulate interpreted by de Broglie relation (according to which circumference of  $n$ -th electron circular orbit contains exactly  $n$  corresponding  $n$ -th de Broglie wave lengths, for  $n = 1, 2, \dots$ ).

According to (4) it follows

$$m_{+n} = n \frac{1}{2\pi R_+} = m_{+1}, \quad \text{for } n = 1, 2, \dots \quad (7)$$

where

$$m_{+1} = \frac{1}{2\pi R_+} \quad (8)$$

Obviously,  $m_{+1}$  depends of  $M$  so that  $m_{+1}$  decreases when  $M$  increases and vice versa. For a macroscopic black hole, i.e. for  $M \gg 1$  it follows  $m_{+1} \ll 1$ .

Now, we shall define the following variable

$$M_+ = S_+ m_{+1} = \frac{A_+}{4} m_{+1} = \frac{1}{2} \frac{R_+^2 + a^2}{R_+} \quad (9)$$

where  $S_+$  represents the Bekenstein-Hawking entropy

$$S_+ = \frac{A_+}{4} = \pi(R_+^2 + a^2). \quad (10)$$

As it is not hard to see it follows

$$M_+ \simeq M \quad \text{for } M^2 \gg a^2 + Q^2 \quad (11)$$

and

$$M_+ = \frac{M}{2} \left(1 + \frac{a^2}{M^2}\right) \geq \frac{M}{2} \quad \text{for } M^2 = a^2 + Q^2. \quad (12)$$

According to (9) it follows

$$S_+ = \frac{M_+}{m_{+1}} \quad (13)$$

which can give a rough intuitive presentation of the black hole entropy. Namely, since according to usual (classical) definition

$$S_+ = \ln N \quad (14)$$

where  $N$  represents the number of the statistical microstates, it follows

$$N = \exp \frac{M_+}{m_{+1}} \quad (15)$$

which represents a rough (classical) estimation of the number of the statistical microstates within given black hole.

Now, we shall differentiate (10) under some additional supposition. Namely, we shall formally, i.e. approximately suppose that by given differentiation all terms that hold  $a$  can be considered almost constant so that their derivations can be neglected. It yields

$$dS_+ = 2\pi R_+ dR_+ \quad (16)$$

Further, it will be supposed that formally, i.e. approximately it is satisfied

$$dR_+ = \left(1 + \frac{M}{(M^2 - a^2 - Q^2)^{\frac{1}{2}}}\right) dM = \frac{R_+}{(M^2 - a^2 - Q^2)^{\frac{1}{2}}} dM \simeq 2dM. \quad (17)$$

Obviously, both (16) and (17) can be satisfied for  $M^2 \gg a^2 + Q^2$ .

Introduction of (17), in (16), yields

$$dS_+ = 2\pi \frac{R_+^2}{(M^2 - a^2 - Q^2)^{\frac{1}{2}}} dM \simeq 4\pi R_+ dM \quad (18)$$

or, in a corresponding finite difference form

$$\Delta S_+ = 4\pi R_+ \Delta M \quad \text{for } \Delta M \ll M. \quad (19)$$

Further, we shall assume

$$\Delta M = nm_{+1} \quad \text{for } n = 1, 2, \quad (20)$$

which, according to (1), (8), yields

$$\Delta S_+ = 2n \quad \text{for } n = 1, 2, \quad (21)$$

that represents the Bekenstein quantization of the black hole entropy. It, according to (10), implies

$$\Delta A_+ = 8n = 2n(2)^2 \quad \text{for } n = 1, 2, \quad (22)$$

that represents the Bekenstein quantization of the black hole surface, where  $2^2$  represents the surface of the quadrate whose side length represents twice Planck length, i.e. 1.

Now we shall attempt to determine  $T_+$  in the following way. We shall consider first thermodynamical law for Kerr-Newman black hole

$$dM = T_+ dS_+ + \Omega_+ dJ + \Phi_+ dQ \quad (23)$$

where

$$\Omega_+ = \frac{a}{R_+^2 + a^2} \quad (24)$$

represents the outer horizon rotation rate, i.e. angular speed, and,

$$\Phi_+ = Q \frac{R_+}{R_+^2 + a^2} \quad (25)$$

- outer horizon electrostatic potential.

Further, we shall approximately neglect term  $\Phi_+ dQ$  in (23) so that this expression turns out in

$$dM = T_+ dS_+ + \Omega_+ dJ \quad (26)$$

According to (3) it follows

$$J = aM \quad (27)$$

so that, according to previous supposition, i.e. approximate condition that  $a$  represents a constant, it follows

$$dJ = a dM. \quad (28)$$

Introduction of accurate form of (17), and (24), (27), (29) in (26) yields the following algebraic equation with unknown variable  $T_+$

$$1 = T_+ 2\pi \frac{R_+^2}{(M^2 - a^2 - Q^2)^{\frac{1}{2}}} dM + \frac{a^2}{R_+^2 + a^2}. \quad (29)$$

It yields

$$T_+ = \frac{1}{2\pi} \frac{(M^2 - a^2 - Q^2)^{\frac{1}{2}}}{R_+^2 + a^2} \quad (30)$$

that represents the Hawking temperature for Kerr-Newman black hole.

In this way we have reproduced, i.e. determined exactly, in the simplest way, three most important thermodynamical characteristics of Kerr-Newman black hole: Bekenstein-Hawking entropy (13), Bekenstein quantization of the black hole entropy (21) or Bekenstein quantization of the black hole surface area (22), and, Hawking temperature (31).

It can be shortly repeated and pointed out that our results are done starting, physically, by assumption that circumference of Kerr-Newman black hole (outer) horizon holds the natural (integer) number of corresponding reduced Compton's wave length, and, mathematically, practically, by simple algebraic equations only. All this is conceptually similar to Bohr's quantization postulate in Bohr's atomic model interpreted by de Broglie relation. Roughly speaking, we gave simply effectively exact predictions on the three well-known most important thermodynamical characteristics of Kerr-Newman black hole (Bekenstein-Hawking entropy, Bekenstein quantization of the black hole entropy or Bekenstein quantization of the black hole surface area, and, Hawking temperature) from a "mesoscopic", like to quasi-classical, view point. But of course, many other important

characteristics of Kerr-Newman black hole cannot be obtained by our simple model. However, our predictions are in the excellent agreement with Copeland and Lahiri work [10]. Namely, Copeland and Lahiri, starting from "microscopic", i.e. string theory, demonstrated that thermodynamical characteristics of the (Schwarzschild) black hole can be obtained by a standing waves corresponding to small oscillations on a circular loop with radius equivalent to (Schwarzschild) horizon radius.

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